

Sl No	Question	Your Answer	Correct Answer	Solution
1	<p>Let A, other than I or $-I$, be a 2×2 real matrix such that $A^2 = I$, I being the unit matrix. Let $\text{Tr}(A)$ be the sum of diagonal elements of A. [Online April 23, 2013]</p> <p>Statement-1: $\text{Tr}(A) = 0$ Statement-2: $\det(A) = -1$</p> <p>(a) Statement-1 is true; Statement-2 is false. (b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. (c) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1. (d) Statement-1 is false; Statement-2 is true.</p>	C	B	<p>(b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$</p> $\begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $b(a+d) = 0, b=0 \text{ or } a=-d \quad \dots(1)$ $c(a+d) = 0, c=0 \text{ or } a=-d \quad \dots(2)$ $a^2 + bc = 1, bc + d^2 = 1 \quad \dots(3)$ <p>'a' and 'd' are diagonal elements $a + d = 0$ statement-1 is correct.</p> <p>Now, $\det(A) = ad - bc$</p> <p>Now, from (3) $a^2 + bc = 1$ and $d^2 + bc = 1$</p> <p>So, $a^2 - d^2 = 0$</p> <p>Adding $a^2 + d^2 + 2bc = 2$ $\Rightarrow (a+d)^2 - 2ad + 2bc = 2$ or $0 - 2(ad - bc) = 2$</p> <p>So, $ad - bc = 1 \Rightarrow \det(A) = -1$</p> <p>So, statement-2 is also true.</p> <p>But statement-2 is not the correct explanation of statement-1.</p>
2	<p>Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define $\text{Tr}(A) = \text{sum of diagonal elements of } A$ and $A = \text{determinant of matrix } A$.</p> <p>Statement - 1 : $\text{Tr}(A) = 0$. Statement - 2 : $A = 1$. [2010]</p> <p>(a) Statement-1 is true, Statement-2 is true : Statement-2 is not a correct explanation for Statement-1. (b) Statement-1 is true, Statement-2 is false. (c) Statement-1 is false, Statement-2 is true. (d) Statement-1 is true, Statement 2 is true ; Statement-2 is a correct explanation for Statement-1.</p>	B	B	<p>(b) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \neq 0$</p> $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$ $\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow a^2 + bc = 1, bc + d^2 = 1$ $ab + bd = ac + cd = 0$ $c \neq 0 \text{ and } b \neq 0 \Rightarrow a + d = 0$ $ A = ad - bc = -a^2 - bc = -1$ <p>Also if $A \neq I$, then $\text{tr}(A) = a + d = 0$.</p> <p>\therefore Statement-1 true and statement-2 false.</p>
3	<p>If B is a 3×3 matrix such that $B^2 = 0$, then $\det[(I+B)^{50} - 50B]$ is equal to: [Online April 9, 2014]</p> <p>(a) 1 (b) 2 (c) 3 (d) 50</p>	C	A	<p>(a) $\det[(I+B)^{50} - 50B]$ $= \det[{}^{50}C_0 I + {}^{50}C_1 B + {}^{50}C_2 B^2 + {}^{50}C_3 B^3 + \dots + {}^{50}C_{50} B^{50} - 50B]$ {All terms having B^n, $2 \leq n \leq 50$ will be zero because given that $B^2 = 0$} $= \det[I + 50B - 50B] = \det[I] = 1$</p>

4	<p>If $1, \omega, \omega^2$ are the cube roots of unity, then</p> $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ <p>is equal to [2003]</p> <p>(a) ω^2 (b) 0 (c) 1 (d) ω</p>	D	B	<p>(b) $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$</p> <p>Expand through R_1</p> $= 1(\omega^{3n} - 1) - \omega^n (\omega^{2n} - \omega^{2n}) + \omega^{2n} (\omega^n - \omega^{4n})$ $= \omega^{3n} - 1 - 0 + \omega^{3n} - \omega^{6n}$ $= 1 - 1 + 1 - 1 = 0 \quad [\because \omega^{3n} = 1]$
5	<p>Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by $\text{tr}(A)$, the sum of diagonal entries of A. Assume that $A^2 = I$. [2008]</p> <p>Statement-1: If $A \neq I$ and $A \neq -I$, then $\det(A) = -1$</p> <p>Statement-2: If $A \neq I$ and $A \neq -I$, then $\text{tr}(A) \neq 0$.</p> <p>(a) Statement-1 is false, Statement-2 is true (b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1 (c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 (d) Statement-1 is true, Statement-2 is false</p>	D	D	<p>(d) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$</p> <p>Given that $A^2 = I$</p> $\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow a^2 + bc = 1 \text{ and } ab + bd = 0$ $ac + cd = 0 \text{ and } bc + d^2 = 1$ <p>From these four equations,</p> $a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$ <p>and $b(a+d) = 0 = c(a+d) \Rightarrow a = -d$</p> $ A = ad - bc = -a^2 - bc = -1$ <p>Also if $A \neq I$ then $\text{tr}(A) = a + d = 0$</p> <p>\therefore Statement 2 is false.</p>
6	<p>The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is non-negative, is : [Online April 10, 2015]</p> <p>(a) $-2\sqrt{2}$ (b) -1 (c) $-16\sqrt{2}$ (d) -8</p>	D	D	<p>(d) $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} \geq 0$</p> $xyz - x - y - z + 2 \geq 0$ $xyz + 2 \geq x + y + z \geq 3(xyz)^{1/3}$ $xyz + 2 - 3(xyz)^{1/3} \geq 0$ $ut(xyz) = t^3$ $t^3 - 3t + 2 \geq 0$ $(t+2)(t-1)^2 \geq 0$ $[t = -2] t^3 = -8$
7	<p>If $f(\theta) = \begin{vmatrix} 1 & \cos \theta & 1 \\ -\sin \theta & 1 & -\cos \theta \\ -1 & \sin \theta & 1 \end{vmatrix}$ and</p> <p>A and B are respectively the maximum and the minimum values of $f(\theta)$, then (A, B) is equal to: [Online April 12, 2014]</p> <p>(a) $(3, -1)$ (b) $(4, 2 - \sqrt{2})$ (c) $(2 + \sqrt{2}, 2 - \sqrt{2})$ (d) $(2 + \sqrt{2}, -1)$</p>	D	C	<p>(c) Let $f(\theta) = \begin{vmatrix} 1 & \cos \theta & 1 \\ -\sin \theta & 1 & -\cos \theta \\ -1 & \sin \theta & 1 \end{vmatrix}$</p> $= (1 + \sin \theta \cos \theta) - \cos \theta(-\sin \theta - \cos \theta) + 1(-\sin^2 \theta + 1)$ $= 1 + \sin \theta \cos \theta + \sin \theta \cos \theta + \cos^2 \theta - \sin^2 \theta + 1$ $= 2 + 2 \sin \theta \cos \theta + \cos 2\theta \dots (1)$ <p>Now, maximum value of (1) is</p> $2 + \sqrt{1^2 + 1^2} = 2 + \sqrt{2}$ <p>and minimum value of (1) is</p> $2 - \sqrt{1^2 + 1^2} = 2 - \sqrt{2}.$

